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Relativistic Quantum Mechanics as a Consequence of the Planck Mass Plasma Conjecture

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Abstract The Planck mass plasma conjecture is the hypothesis that the vacuum of space is a kind of plasma composed of positive and negative Planck mass particles interacting by the Planck force over a Planck length, repulsive for equal and attractive for unequal Planck masses. The hypothesis permits to derive quantum mechanics and Lorentz invariance as asymptotic approximations for energies small compared to the Planck energy. Besides a spectrum of elementary particles greatly resembling the particles of the standard model, the hypothesis gives a value of the fine structure constant at the energy where the strong, the weak, and electromagnetic interaction become equal.

1 Introduction

String (resp. M) theory unification attempts are characterized by going to ever larger groups and higher dimensions. Very different, but at the present time much less pursued attempts are guided by analogies between condensed matter and elementary particle physics, and analogies between condensed matter physics and general relativity [1–9]. These analogies suggest that the fundamental group of nature is small, with higher groups and Lorentz invariance derived from the dynamics at a more fundamental level. A likewise dynamic reduction of higher symmetries to an underlying simple symmetry is known from crystal physics, where the large number of symmetries actually observed are reduced to the spherical symmetry of the Coulomb field.

We make here the proposition that the fundamental group is SU2, and that by Planck's conjecture the fundamental equations of physics contain as free parameters only the Planck length r_p, the Planck mass m_p and Planck time t_p (G Newton's constant, h Planck's con-

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stant, c the velocity of light).

$$
\begin{aligned}
r_p &= \sqrt{\frac{\hbar G}{c^3}} \approx 10^{-33}\ \text{cm},\\
m_p &= \sqrt{\frac{\hbar c}{G}} \approx 10^{-5}\ \text{g},\\
t_p &= \sqrt{\frac{\hbar G}{c^5}} \approx 10^{-44}\ \text{s}.
\end{aligned}
\tag{1.1}
$$

The assumption that SU2 is the fundamental group means nature works like a computer with a binary number system. As noted by von Weizsäcker [10] with SU2 isomorphic to SO3, the rotation group in R3, then immediately explains why natural space is three-dimensional.

2 Planck Mass Plasma

The Planck mass plasma conjecture is the assumption that the vacuum of space is densely filled with an equal number of positive and negative Planck mass particles, with each Planck length volume in the average occupied by one Planck mass, with the Planck mass particles interacting with each other by the Planck force over a Planck length, and with Planck mass particles of equal sign repelling and those of opposite sign attracting each other.[1] The particular choice made for the sign of the Planck force is the only one which keeps the Planck mass plasma stable. While Newton's action = reaction remains valid for the interaction of equal Planck mass particles, it is violated for the interaction of a positive with a negative Planck mass particle, even though globally the total linear momentum of the Planck mass plasma is conserved, with the recoil absorbed by the Planck mass plasma as a whole.

It is the local violation of Newton's actio = reactio which leads to quantum mechanics at the most fundamental level, as can be seen as follows: Under the Planck force $F_p = m_p c^2/r_p$, the velocity fluctuation of a Planck mass particle interacting with a Planck mass particle of opposite sign is $\Delta v = (F_p/m_p)t_p = (c^2/r_p)(r_p/c) = c$, and hence the momentum fluctuation $\Delta p = m_p c$. But since $\Delta q = r_p$, and because $m_p r_p c = \hbar$, one obtains Heisenberg's uncertainty relation $\Delta p \Delta q = \hbar$, for a Planck mass particle. Accordingly, the quantum fluctuations are explained by the interaction with hidden negative masses, with energy borrowed from the sea of hidden negative masses.

The conjecture that quantum mechanics has its cause in the interaction of positive with hidden negative masses is supported by its derivation from a variational principle first proposed by Fenyes [11]. Because Fenyes could not give a physical explanation for his variational principle he was criticized by Heisenberg [12], but the Planck mass plasma hypothesis gives a simple explanation through the existence of negative masses.

According to Newtonian mechanics and Planck's conjecture, the interaction of a positive with a negative Planck mass particle leads to a velocity fluctuation $\dot{\delta} = a_p t_p = c$, with a displacement of the particle equal to $\delta = (1/2)a_p t_p^2 = r_p/2$, where $a_p = F_p/m_p$. Therefore,

[1] It was shown by Planck in 1911 that there must be a divergent zero point vacuum energy by Nernst called an aether. It is for this reason that I have also called Planck's zero point vacuum energy the Planck aether. Calling it a Planck mass plasma instead appears possible as well.

a Planck mass particle immersed in the Planck mass plasma makes a stochastic quivering motion (Zitterbewegung) with the velocity

$$\mathbf{v}_D = -(r_p c/2)(\nabla n/n) \tag{2.1}$$

where on average $n = 1/2r_p^3$ is the number density of positive or negative Planck mass particles. The kinetic energy of this diffusion process is given by

$$\left(\frac{m_p}{2}\right)v_D^2 = \left(\frac{m_p}{8}\right)r_p^2 c^2\left(\frac{\nabla n}{n}\right)^2 = \left(\frac{\hbar^2}{8m_p}\right)\left(\frac{\nabla n}{n}\right)^2. \tag{2.2}$$

Putting

$$\mathbf{v} = \frac{\hbar}{m_p}\nabla S \tag{2.3}$$

where S is the Hamilton action function and v the velocity of the Planck mass plasma, the Lagrange density for the Planck mass plasma is

$$\mathbf{L} = n\left[\hbar\frac{\partial S}{\partial t} + \frac{\hbar^2}{2m_p}(\nabla S)^2 + U + \frac{\hbar^2}{8m_p}\left(\frac{\nabla n}{n}\right)^2\right]. \tag{2.4}$$

Variation of (2.4) with regard to S according to

$$\frac{\partial}{\partial t}\left(\frac{\partial \mathbf{L}}{\partial S/\partial t}\right) + \frac{\partial}{\partial \mathbf{r}}\left(\frac{\partial \mathbf{L}}{\partial S/\partial \mathbf{r}}\right) = 0 \tag{2.5}$$

leads to

$$\frac{\partial n}{\partial t} + \frac{\hbar}{m_p}\nabla\cdot(n\nabla S) = 0 \tag{2.6}$$

or

$$\frac{\partial n}{\partial t} + \nabla\cdot(n\mathbf{v}) = 0 \tag{2.7}$$

which is the continuity equation of the Planck mass plasma. Variation with regard to n according to

$$\frac{\partial \mathbf{L}}{\partial n} - \frac{\partial}{\partial \mathbf{r}}\left(\frac{\partial \mathbf{L}}{\partial n/\partial \mathbf{r}}\right) = 0 \tag{2.8}$$

leads to

$$\hbar\frac{\partial S}{\partial t} + U + \frac{\hbar^2}{2m_p}(\nabla S)^2 + \frac{\hbar^2}{4m_p}\left[\frac{1}{2}\left(\frac{\nabla n}{n}\right)^2 - \frac{\nabla^2 n}{n}\right] = 0 \tag{2.9}$$

or

$$\hbar\frac{\partial S}{\partial t} + U + \frac{\hbar^2}{2m_p}(\nabla S)^2 + \frac{\hbar^2}{2m_p}\frac{\nabla^2\sqrt{n}}{\sqrt{n}} = 0 \tag{2.10}$$

with the Madelung transformation

$$\begin{cases} \psi = \sqrt{n}e^{iS}, \\ \psi^* = \sqrt{n}e^{-iS} \end{cases} \tag{2.11}$$

(2.6) and (2.10) is obtained from the Schrödinger equation

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m_p}\nabla^2\psi + U\psi. \tag{2.12}$$

For the potential U, coming from all the Planck mass particles acting on one Planck mass particle $\pm m_p$ described by the wavefunction $\psi_\pm$, we set

$$U_\pm = \pm 2\hbar c r_p^2 \langle \psi_\pm^* \psi_\pm - \psi_\mp^* \psi_\mp \rangle \tag{2.13}$$

which we justify as follows: A dense assembly of positive and negative Planck mass particles, each of them occupying the volume r_p^3, has the expectation value $\langle \psi_\pm^* \psi_\pm \rangle = 1/2r_p^3$, whereby $2\hbar c r_p^2 \langle \psi_\pm^* \psi_\pm \rangle = m_p c^2$, implying an average potential $\pm m_p c^2$ for the positive or negative Planck mass particles within the Planck mass plasma and consistent with $F_p r_p = m_p c^2$. We thus have for both the positive and negative Planck mass particles:

$$i\hbar\frac{\partial\psi_\pm}{\partial t} = \mp\frac{\hbar^2}{2m_p}\nabla^2\psi_\pm \pm 2\hbar c r_p^2 \langle \psi_\pm^* \psi_\pm - \psi_\mp^* \psi_\mp \rangle \psi_\pm. \tag{2.14}$$

To make the transition from the one particle Schrödinger equation (2.14) to the many particle equation of the Planck mass plasma we replace (2.14) by

$$i\hbar\frac{\partial\psi_\pm}{\partial t} = \mp\frac{\hbar^2}{2m_p}\nabla^2\psi_\pm \pm 2\hbar c r_p^2 (\psi_\pm^\dagger \psi_\pm - \psi_\mp^\dagger \psi_\mp) \psi_\pm \tag{2.15}$$

where $\psi_\pm^\dagger$, $\psi_\pm$ are field operators satisfying the commutation relations

$$\begin{cases} [\psi_\pm(\mathbf{r}), \psi_\pm^\dagger(\mathbf{r}')] = \delta(\mathbf{r}-\mathbf{r}'), \\ [\psi_\pm(\mathbf{r}), \psi_\pm(\mathbf{r}')] = [\psi_\pm^\dagger(\mathbf{r}), \psi_\pm^\dagger(\mathbf{r}')] = 0. \end{cases} \tag{2.16}$$

Replacing in (2.15) $\psi_\pm^\dagger$, $\psi_\pm$ by the classical field functions $\varphi_\pm^*$, $\varphi_\pm$, (2.15) becomes a nonlinear Schrödinger equation which can be derived from the Lagrange density

$$\mathbf{L}_\pm = i\hbar\varphi_\pm^*\dot{\varphi}_\pm \mp \frac{\hbar^2}{2m_p}(\nabla\varphi_\pm^*)(\nabla\varphi_\pm) \mp 2\hbar c r_p^2\left[\frac{1}{2}\varphi_\pm^*\varphi_\pm - \varphi_\mp^*\varphi_\mp\right]\varphi_\pm^*\varphi_\pm \tag{2.17}$$

with the Hamilton density

$$\mathbf{H}_\pm = \pm\frac{\hbar^2}{2m_p}(\nabla\varphi_\pm^*)(\nabla\varphi_\pm) \pm 2\hbar c r_p^2\left[\frac{1}{2}\varphi_\pm^*\varphi_\pm - \varphi_\mp^*\varphi_\mp\right]\varphi_\pm^*\varphi_\pm. \tag{2.18}$$

With $H_\pm = \int \mathbf{H}_\pm d\mathbf{r}$ one has the Heisenberg equation of motion

$$i\hbar\dot{\psi}_\pm = [\psi_\pm, H_\pm] \tag{2.19}$$

which agrees with (2.15) and shows that the classical and quantum equations have the same form. For the particle number operator $N_\pm = \int \psi_\pm^\dagger \psi_\pm d\mathbf{r}$ one finds that

$$i\hbar\dot{N}_\pm = [N_\pm, H_\pm] = 0 \tag{2.20}$$

which shows that the particle numbers are conserved, permitting the permanent existence of negative masses.

3 Formation of a Vortex Lattice

In a frictionless fluid the Reynolds number is infinite. In the presence of some internal motion the flow is unstable decaying into vortices. By comparison, the superfluid Planck mass plasma is unstable even in the absence of any internal motion, because with an equal number of positive and negative mass particles it can without the expenditure of energy by spontaneous symmetry breaking decay into a vortex sponge (resp. vortex lattice), of quantized vortices with the vortex core radius equal the Planck length where the velocity reaches the velocity of light.

In nonquantized fluid dynamics the vortex core radius is about equal a mean free path λ, where the velocity reaches the velocity of sound, the latter about equal the thermal molecular velocity v_t. With the kinematic viscosity $\nu \approx v_t \lambda$, the Reynolds number in the vortex core is

$$Re = vr/\nu = 1. \tag{3.1}$$

Interpreting Schrödinger's equation as an equation with an imaginary quantum viscosity $\nu_Q = i\hbar/2m_p \sim i r_p c$, and defining a quantum Reynolds number

$$Re^Q = i v r/\nu_Q \tag{3.2}$$

one finds with $\nu_Q \sim i r_p c$, that in the vortex core of a quantum fluid (where $v = c$), $Re^Q \sim 1$. Because of this analogy one can apply the stability analysis for a lattice of vortices in nonquantum fluid dynamics to a vortex lattice in quantum fluid dynamics. For the two-dimensional Karman vortex street the stability was analyzed by Schlayer [13], who found that the radius r_0 of the vortex core must be related to the distance ℓ between two vortices by

$$r_0 \approx 3.4 \times 10^{-3} \ell. \tag{3.3}$$

Setting $r_0 = r_p$ and $\ell = 2R$, where R is the radius of a vortex lattice cell occupied by one line vortex, one has

$$\frac{R}{r_p} \approx 147. \tag{3.4}$$

No comparable stability analysis seems to have been performed for a three-dimensional vortex lattice made up of vortex rings. The instability leading to the decay into vortices apparently arises from the disturbance one vortex exerts on an adjacent vortex. At the distance R/r_p the velocity by a vortex ring is larger by the factor $\log(8R/r_p)$, compared to the velocity of a line vortex at the same distance. With $R/r_p \approx 147$ for a line vortex lattice, a value of R/r_p for a ring vortex lattice can be estimated by solving for R/r_p the equation

$$R/r_p \approx 147 \log\left(\frac{8R}{r_p}\right) \tag{3.5}$$

and one finds that

$$R/r_p \approx 1360. \tag{3.6}$$

As was shown by the author [14] the value of the fine structure constant at the energy where the strong, weak, and electromagnetic constant become equal can be explained in terms of this ratio by

$$\frac{1}{\alpha} = \frac{12\pi}{11} \log\left(\frac{R}{r_p}\right). \tag{3.7}$$

For $R/r_p = 1360$, one finds that $1/\alpha = 24.8$, in surprisingly good agreement with the empirical value, $1/\alpha = 25$.

4 The Origin of Charge

Through their zero point fluctuations Planck mass particles bound in vortex filaments have a kinetic energy density by order of magnitude equal to

$$\varepsilon \approx \frac{m_p c^2}{r_p^3} = \frac{\hbar c}{r_p^4}. \tag{4.1}$$

By order of magnitude this is about equal the energy density g^2, where g is the Newtonian gravitational field of a Planck mass particle m_p at the distance $r = r_p$. Because with

$$g \approx \frac{\sqrt{G} m_p}{r_p^2} \tag{4.2}$$

one has

$$g^2 \approx \frac{G m_p^2}{r_p^4} = \frac{\hbar c}{r_p^4}. \tag{4.3}$$

The interpretation of this result is as follows: Through its zero-point fluctuations a Planck mass particle bound in a vortex filament becomes the source of virtual phonons setting up a Newtonian type attractive force field with the coupling constant $G m_p = \hbar c$. Charge is thus explained by the zero point fluctuations of the Planck mass particles bound in vortices. The smallness of the gravitational coupling constant is explained by the near cancellation of the kinetic energy coming from the positive and negative mass component of the Planck mass plasma. Such a cancellation does not happen for fields not coupled to the energy momentum tensor. Therefore, with the exception of the gravitational coupling constant, all other coupling constants are within a few order of magnitude equal to $\hbar c$ [15].

5 Spectrum of Quasiparticles

The quantized modes of the vortex lattice give a spectrum of quasiparticles, bosonic and fermionic. The spectrum of bosonic quasiparticles contains gravitons and photons as the symmetric and antisymmetric wave modes of the vortex lattice. The spectrum of fermionic quasiparticles contains electrons and neutrinos with a maximum of four families. Quarks and gluons can be explained as vortex substructures. For the fermionic quasiparticles the existence of negative masses is essential, because fermions can be understood as gravitationally bound positive-negative mass "pole-dipole" particles with Schrödinger's "Zitterbewegung", leading to the Dirac equation and a spectrum of fermions greatly resembling the spectrum of fermions in the standard model [16].

The purpose of this communication is to show that the Planck mass plasma conjecture leads to Lorentz invariance of these quasiparticles, as in the dynamic pre-Einstein theory of relativity by Lorentz and Poincare. The reader who is interested in the other aspects of the theory is referred to reference [2] and to the recently published monograph by the author [16].

6 Quantum Mechanics and Lorentz Invariance

According to Einstein and Hopf, the friction force acting on a charged particle moving with velocity v through an electromagnetic radiation field with a frequency dependent spectrum $f(\omega)$ is given by [17]

$$F = -\text{const} \cdot \left[f(\omega) - \frac{\omega}{3} \frac{df(\omega)}{d\omega} \right] v. \tag{6.1}$$

This force vanishes if

$$f(\omega) = const \cdot \omega^3. \tag{6.2}$$

It is plausible that this is universally true, not only for electromagnetic interactions. But now, the spectrum (6.2) not only is frictionless, but it is also Lorentz invariant.

In the Planck mass plasma the zero point energy results from the "Zitterbewegung" caused by the interaction of positive with negative Planck mass particles. To relax it into a frictionless state as is the case for a superfluid, the spectrum must assume the form (6.2). With $4\pi\omega^2 d\omega$ modes of oscillation in between ω and $\omega + d\omega$ the energy for each mode must be proportional to ω to obtain, as in quantum mechanics, the ω^3 dependence (6.2). Because the spectrum (6.2) is generated by collective oscillations of the discrete Planck mass particles, it has to be cut off at the Planck frequency $\omega_p = c/r_p = 1/t_p$, where the zero point energy is equal to $(1/2)\hbar\omega_p = (1/2)m_p c^2$. It thus follows that the zero point energy of each mode with a frequency $\omega < \omega_p$ must be $E = (1/2)\hbar\omega$. A cut-off of the zero point energy at the Planck frequency destroys Lorentz invariance, but only for frequencies near the Planck frequency and hence only at extremely high energies. The nonrelativistic Schrödinger equation, in which the zero point energy is expressed through the kinetic energy term $-(\hbar^2/2m)\nabla^2\psi$, therefore remains valid for masses $m < m_p$, to be replaced by Newtonian mechanics for masses $m > m_p$.

A cut-off at the Planck frequency generates a distinguished reference system in which the zero point energy spectrum is isotropic and at rest. In this distinguished reference system, the scalar potential from which the forces are to be derived satisfies the inhomogeneous wave equation:

$$-\frac{1}{c^2}\frac{\partial^2 \Phi}{\partial t^2} + \nabla^2 \Phi = -4\pi\rho(r,t) \tag{6.3}$$

where $\rho(r,t)$ are the sources of this field. For a body in static equilibrium at rest in the distinguished reference system for which the sources are those of the body itself one has

$$\nabla^2 \Phi = -4\pi\rho(r). \tag{6.4}$$

If set into absolute motion with the velocity v along the x-axis, the coordinates of the reference system at rest with the moving body are obtained by the Galilei transformation:

$$x' = x - vt, \qquad y' = y, \qquad z' = z, \qquad t' = t \tag{6.5}$$

transforming (6.3) into

$$-\frac{1}{c^2}\frac{\partial^2 \Phi'}{\partial t'^2} + \frac{2v}{c^2}\frac{\partial^2 \Phi'}{\partial x' \partial t'} + \left(1 - \frac{v^2}{c^2}\right)\frac{\partial^2 \Phi'}{\partial x'^2} + \frac{\partial^2 \Phi'}{\partial y'^2} + \frac{\partial^2 \Phi'}{\partial z'^2} = 4\pi\rho(\mathbf{r}', t'). \tag{6.6}$$

After the body has settled into a new equilibrium in which $\partial/\partial t' = 0$, one has instead of (6.4):

$$\left(1 - \frac{v^2}{c^2}\right)\frac{\partial^2 \Phi'}{\partial x'^2} + \frac{\partial^2 \Phi'}{\partial y^2} + \frac{\partial^2 \Phi'}{\partial z^2} = -4\pi\rho(x', y, z). \tag{6.7}$$

Comparison of (6.7) with (6.4) shows that the l.h.s. of (6.7) is the same is one sets $\Phi' = \Phi$ and $dx' = dx\sqrt{1 - v^2/c^2}$. This implies a uniform contraction of the body by the factor $\sqrt{1 - v^2/c^2}$, because the sources within the body are contracted by the same factor, whereby the r.h.s. of (6.7) becomes equal to the r.h.s. of (6.4). Since all clocks can be viewed as light clocks (made up from rods with light signals reflected back and forth along the rod), the length contraction leads to slower going clocks, going slower by the same factor. Thus using contracted rods and slower going clocks as measuring devices, (6.3) is Lorentz invariant.

With the repulsive quantum force having its cause in the zero point energy and with the zero point energy Lorentz invariant, it follows that the attractive electric force is balanced by the repulsive quantum force always in such a way that Lorentz invariance appears to be true. Departures from Lorentz invariance could then only be noticed for particle energies near the Planck energy, where Lorentz invariance is violated through the cut-off of the zero point energy spectrum.

With Dirac spinors explained as excitonic quasiparticles held together by zero rest mass bosonic force fields [16], the foregoing applies to all fermionic elementary particles which for this reason can be described by relativistic quantum field theory.

7 Quantum Mechanical Nonlocality

According to the Copenhagen interpretation, the wave function is only an expression of our knowledge and any attempt to explain the nonlocal effects by some underlying "hidden parameters" is doomed to fail. With the Copenhagen interpretation ultimately requiring conscious observers, absent from most places of the physical universe, not all physicists have adopted the Copenhagen interpretation as gospel truth. The most widely known attempt for a different interpretation is the pilot wave theory by de Broglie and Bohm, viewing the Schrödinger wave as a guiding field for the particle motion. Apart from its violation of Newton's actio = reactio, with the particle guided by the wave not exerting a recoil on the wave, the pilot wave theory leaves unanswered the question of the physical character of the wave. In keeping the whole mathematical structure of quantum theory, the pilot wave theory was called by Einstein as "too cheap". With physics having its origin in very high energies near the Planck energy, attempts to understand quantum mechanics should be undertaken at these high energies. In the Planck mass plasma conjecture, physics has its foundation in the Newtonian mechanics of positive and negative Planck masses, locally interacting over short distances. The nonlocal effects of quantum mechanics should for this reason be explained by the local interaction of all the Planck masses. In the Planck mass plasma all particles, save and except the Planck mass particles themselves, are quasiparticles, that is they are collective modes involving a very large number of Planck mass particles. The occurrence of nonlocal (resp. with infinite velocity) transmitted interactions is not so uncommon to classical physics. In the theory of incompressible fluid dynamics, pressure forces are transmitted with infinite speed, giving the wrong physical perception of nonlocality. The nonlocality of incompressible fluid dynamics is a result of the mathematical model of an incompressible fluid which in reality is an approximation, valid for velocities small compared to the velocity of sound. The same argument can be made for quantum mechanics, if one recognizes quantum mechanics as a model-dependent approximation. With the Planck energy so very much larger than the energy scale of everyday life, quantum mechanics would remain a very good approximation. The superluminal nonlocal quantum mechanical actions are perhaps the strongest case for a background medium which may have wave modes with a phase velocity exceeding the velocity of light. Because the nonlocal actions of quantum mechanics cannot transmit any information, the existence of superluminal phase velocities is all that is needed.

We will now show that with the Planck mass plasma conjecture, many of the mysteries surrounding quantum mechanics don't look so mysterious after all. If physics has its root at the extremely high Planck energy, it should be of no surprise that effects projected from this energy scale down to the energy scale of everyday life may look incomprehensible.

8 Sagnac and Aharonov Bohm Effect

A good example for quantum mechanical nonlocality is the Aharonov-Bohm effect, and it is instructive to compare it with the Sagnac effect. The outcome of his rotating interferometer experiment was used by Sagnac as a decisive argument against Einstein's claim that physics could do without the aether hypothesis [18]. Most text-books treating the Sagnac effect, like the well–known theoretical physics course by Sommerfeld [19], explain the effect by Sagnac's original argument, that it is caused by a whirling motion of the aether felt in a rotating reference system, but it is also often stated that the effect somehow is caused by the centrifugal- and Coriolis-forces and for this reason should be treated in the framework of general relativity. A discussion of the Sagnac effect within general relativity can be found in the theoretical physics lectures by Landau and Lifshitz [20], where it is alleged that the effect is caused by the special stationary gravitational field set up in a rotating reference frame, even though there are no masses present as the source of a true gravitational field. Already Ives [21], had given convincing arguments that the effect has nothing to do with the inertial forces in a rotating reference frame, keeping Sagnac's original argument to be valid as ever.

The interpretation of the Sagnac effect, which says that it is caused by an aether wind in the rotating reference system, would make in this system the velocity of light anisotropic. Such a conclusion seems to contradict the outcome of the Michelson–Morley and other experiments which claim to have proven the constancy of the velocity of light, which is why Einstein discarded the aether hypothesis. A close examination of all these experiments, however, shows that one there never measures the one-way velocity of light, but rather always the to and fro velocity. For this reason only the scalar c^2, not the vector $\mathbf{c}$, enters into the Lorentz transformation formulas, and Sagnac's interpretation is not subject to any logical contradiction if the dynamic interpretation of special relativity is adopted.

An effect where a phase shift on an electron wave occurs in the absence of any electromagnetic forces has been described by Aharonov and Bohm [22]. There, a change in the magnetic vector potential alone can produce a shift. The effect is explained as a direct consequence of Schrödinger's wave equation, into which the potentials and not the force fields enter. For this reason a case was made by Aharonov and Bohm to give the potentials a more direct physical meaning rather than to be just a convenient mathematical tool. However, to elevate the potentials to true physical significance has the problem that these potentials can always be changed by a gauge transformation without affecting the physical results derived from them. Because the potentials can measurably influence the phase of an electron wave in the absence of any electromagnetic force fields, it has alternatively been claimed that the effect is a proof for action at a distance in quantum mechanics, where the magnetic force field inside a magnetic solenoid can influence the phase of the electron wave in the force-free region outside the solenoid. Instead we will show that both the Sagnac and the Aharonov–Bohm effect can be understood to result from a rotational aether motion, revealing a close relationship between these effects. This picture not only gives a full physical explanation of the potentials and the meaning of a gauge transformation but also eliminates the need for any hypothetical action at a distance.

In the Sagnac effect, a light beam is split into two separate beams, each one following a half circle of radius r along the periphery of a rotating table. One of the two beams is propagating in the same direction as the velocity of the rotating table, the other one in the opposite direction. The rotating table shall have the angular velocity $-\Omega$, making its velocity at the radius r equal to $u = -r\Omega$.

According to the hypothesis of an aether at rest in the unaccelerated laboratory, the velocity of light judged from a co-rotating reference system would be $c - v$ for the beam propagating in the same direction as the rotating table and $c + v$ for the beam propagating in the opposite direction, where $v = -u$ is the aether velocity in the rotating reference system. The time difference for both beams leaving from the position A and arriving at position B follows from

$$c\delta t = \int_A^B (c+v)dt - \int_A^B (c-v)dt \tag{8.1}$$

hence

$$\delta t = (1/c)\oint v dt. \tag{8.2}$$

If we are only interested in first order effects in v/c, we can put $dt \approx (1/c)ds$, where ds is the line element along the circular path. If the light propagates along an arbitrary but simply connected curve we then have

$$\delta t = (1/c^2)\oint \mathbf{v}\cdot d\mathbf{s} \tag{8.3}$$

for which we can also write

$$\delta t = (1/c^2)\oint \operatorname{curl}\mathbf{v}\cdot d\mathbf{f} \tag{8.4}$$

where $F = |\int d\mathbf{f}|$ is the surface enclosed by the light path. Because $\Omega = (1/2)|\operatorname{curl}\mathbf{v}|$ we find

$$\delta t = 2\Omega F/c^2 \tag{8.5}$$

which agrees with the result obtained by Sagnac.

For the phase shift $\delta\phi = \omega\delta t$, where ω is the circular frequency of the wave, we have

$$\delta\phi = (\omega/c^2)\oint \mathbf{v}\cdot d\mathbf{s}. \tag{8.6}$$

In Maxwell's equations the electric and magnetic fields can be expressed through a scalar potential Φ and a vector potential $\mathbf{A}$:

$$\begin{aligned} \mathbf{E} &= -\frac{1}{c}\frac{\partial\mathbf{A}}{\partial t} - \operatorname{grad}\Phi, \\ \mathbf{H} &= \operatorname{curl}\mathbf{A} \end{aligned} \tag{8.7}$$

E and **H** remain unchanged under the gauge transformation of the potentials

$$\begin{aligned}\Phi' &= \Phi - \frac{1}{c}\frac{df}{dt},\\ \mathbf{A}' &= \mathbf{A} + \operatorname{grad} f\end{aligned} \tag{8.8}$$

where f is called the gauge function. Imposing on Φ and $\mathbf{A}$ the Lorentz gauge condition

$$\frac{1}{c}\frac{\partial \Phi}{\partial t} + \operatorname{div}\mathbf{A} = 0 \tag{8.9}$$

the gauge function must satisfy the wave equation

$$-\frac{1}{c^2}\frac{\partial^2 f}{\partial t^2} + \nabla^2 f = 0. \tag{8.10}$$

In an electromagnetic field the force on a charge e is

$$\mathbf{F} = e\left[\mathbf{E} + \frac{1}{c}\mathbf{v}\times\mathbf{H}\right] = \left[-\frac{1}{c}\frac{\partial \mathbf{A}}{\partial t} - \operatorname{grad}\Phi + \frac{1}{c}\mathbf{v}\times\operatorname{curl}\mathbf{A}\right]. \tag{8.11}$$

By making a gauge transformation of the Hamilton operator in the Schrödinger wave equation, the wave function transforms as

$$\psi' = \psi \exp\left[\frac{ie}{\hbar c} f\right] \tag{8.12}$$

leaving invariant the probability density $\psi^*\psi$.

To give gauge invariance a hydrodynamic interpretation, we compare (8.11) with the force acting on a test body of mass m placed into the moving Planck mass plasma. This force follows from Euler's equation and is

$$F = m\frac{d\mathbf{v}}{dt} = m\left[\frac{\partial \mathbf{v}}{\partial t} + \operatorname{grad}\left(\frac{\mathbf{v}^2}{2}\right) - \mathbf{v}\times\operatorname{curl}\mathbf{v}\right]. \tag{8.13}$$

Complete analogy between (8.11) and (8.13) is established if one sets

$$\begin{aligned}\Phi &= -\tfrac{m}{2e}\mathbf{v}^2,\\ \mathbf{A} &= -\tfrac{mc}{e}\mathbf{v}.\end{aligned} \tag{8.14}$$

According to (8.2) and (8.6) Φ and $\mathbf{A}$ shift the phase of a Schrödinger wave by

$$\begin{aligned}\delta\phi &= \frac{e}{\hbar}\int_{t_1}^{t_2}\Phi\, dt,\\ \delta\phi &= \frac{e}{\hbar c}\oint \mathbf{A}\cdot d\mathbf{s}.\end{aligned} \tag{8.15}$$

To make a comparison with the Sagnac effect, we consider the magnetic field produced by an infinitely long cylindrical solenoid of radius R. Inside the solenoid the field is constant, vanishing outside. If the magnetic field inside the solenoid is H, the vector potential is (magnetic field directed downwards):

$$A_\phi = \begin{cases} -\frac{1}{2}Hr, & r < R, \\ -\frac{1}{2}\frac{HR^2}{r}, & r > R. \end{cases} \tag{8.16}$$

According to (8.15) the vector potential on a closed path leads to the phase shift

$$\delta\phi = \begin{cases} -\frac{e}{\hbar c}H\pi r^2, & r < R, \\ -\frac{e}{\hbar c}H\pi R^2, & r > R. \end{cases} \tag{8.17}$$

As noted by Aharonov and Bohm [22], there is a phase shift for $r > R$, even though $H = 0$ (because for $r > R$, $\operatorname{curl}\mathbf{A} = 0$).

Expressing $\mathbf{A}$ by (8.14) through v, the hypothetical circular aether velocity, one has

$$v_\phi = \begin{cases} \frac{e}{2mc}Hr, & r < R, \\ \frac{e}{2mc}\frac{HR^2}{r} & r > R. \end{cases} \tag{8.18}$$

One sees that inside the coil the velocity profile is the same as in a rotating frame of reference, having outside coil the form of a potential vortex. If expressed in terms of the aether velocity, the phase shift becomes

$$\delta\phi = \frac{m}{\hbar}\oint \mathbf{v}\cdot d\mathbf{s} \tag{8.19}$$

with $\hbar\omega = mc^2$ this is the same as (8.6) for the Sagnac effect.

For the magnetic vector potential the aether velocity can easily become much larger than in any rotating platform experiment. According to (8.19) the velocity reaches a maximum at $r = R$, where it is

$$\frac{|v_{\max}|}{c} = \frac{eHR}{2mc^2}. \tag{8.20}$$

For electrons this is $|v_{\max}|/c \approx 3\times 10^{-4}HR$, where H is measured in Gauss. Assuming that $H = 10^4$ G, this would mean that $|v_{\max}| > c$ for $R \gtrsim 0.3$ cm. It thus seems to follow that the aether can reach superluminal velocities for rather modest magnetic fields. In this regard it must be emphasized that in the Planck mass plasma all relativistic effects are explained dynamically, with the aether, which is here the Planck mass plasma, obeying an exactly nonrelativistic law of motion. It can for this reason assume superluminal velocities, and if this would be the same velocity felt on a rotating platform, it would lead to an enormous centrifugal and Coriolis-field inside the coil, obviously not observed.

The Planck mass plasma can give a simple explanation for this paradox because it consists of two superfluid components, one composed of positive Planck masses and the other one of negative Planck masses. The two components can freely flow through each other making possible two configurations, one where both components are co-rotating, and one where they are counter-rotating. The co-rotating configuration is obviously realized on a rotating platform where it leads to the Sagnac and neutron interference effects. This suggests that in the presence of a magnetic vector potential the two superfluid components are counter-rotating. Outside the coil where curl $\mathbf{A} = 0$ the magnetic energy density vanishes, implying that the magnitude of both velocities are exactly the same. Inside the coil where curl $\mathbf{A} \neq 0$, there must be a small imbalance in the velocity of the positive over the negative Planck masses to result in a positive energy density.

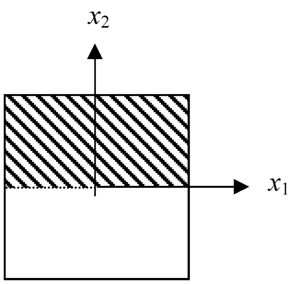
Whereas in the Sagnac effect $\omega = \text{curl}\,\mathbf{v} \neq 0$, according to (8.13) resulting in observable inertial forces, no magnetic forces are present in the Aharonov–Bohm effect where curl $\mathbf{A} = 0$. In the Sagnac effect the aether makes a uniform rotational motion whereas in the Aharonov–Bohm effect, the aether motion is a potential vortex. With the kinetic energy of the positive Planck masses cancelled by the kinetic energy of the negative Planck masses, the energy of the vortex is zero. Since for shifting the phase no energy is needed, there can be no contradiction.

9 Entanglement

A more serious problem of quantum mechanical nonlocality is the strange entanglement of the many body Schrödinger equation in configuration space. In the Planck mass plasma this problem is avoided because it views all particles as quasiparticles of this plasma, and it is incorrect to visualize a many-body wave function to be composed of the same particles which are observed before an interaction in between the particles is turned on. In the presence of an interaction it rather leads to a new set of quasiparticles into which the wave function can be factorized. This can be demonstrated for two identical particles moving in a harmonic oscillator well. The well shall have its coordinate origin at $x = 0$, with the first particle having the coordinate x_1 and the second on the coordinate x_2. Considering two oscillator wave functions $\psi_0(x)$ and $\psi_1(x)$, with ψ_0 having no node and $\psi_1(x)$ having one node, there are two two-particle wave functions:

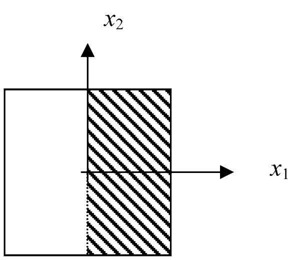
$$\psi(x_1, x_2) = \psi_0(x_1)\psi_1(x_2) = \sqrt{\frac{2}{\pi}} x_2 e^{-(x_1^2+x_2^2)/2}$$

$$= \qquad (9.1a)$$



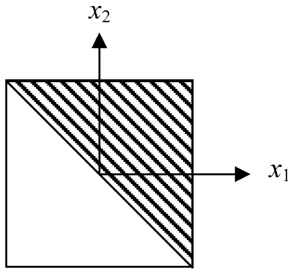
$$\psi(x_1, x_2) = \psi_1(x_1)\psi_0(x_2) = \sqrt{\frac{2}{\pi}} x_1 e^{-(x_1^2+x_2^2)/2}$$

$$= \qquad (9.1b)$$



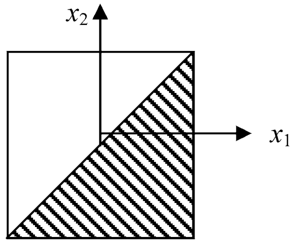
graphically displayed in the x_1, x_2 configuration space, with the nodes along the lines $x_2 = 0$ and $x_1 = 0$. By a linear superposition of these wave functions we get a symmetric and an antisymmetric combination:

$$\psi_s(x_1, x_2) = \frac{1}{\sqrt{2}}[\psi_0(x_1)\psi_1(x_2) + \psi_1(x_1)\psi_0(x_2)] = \frac{1}{\sqrt{\pi}}(x_2 + x_1)e^{-(x_1^2+x_2^2)/2}$$

= 

(9.2a)

$$\psi_a(x_1, x_2) = \frac{1}{\sqrt{2}}[\psi_0(x_1)\psi_1(x_2) - \psi_1(x_1)\psi_0(x_2)] = \frac{1}{\sqrt{\pi}}(x_2 - x_1)e^{-(x_1^2+x_2^2)/2}$$

= 

(9.2b)

If a perturbation is applied whereby the two particles slightly attract each other, the degeneracy for the two wave–functions is removed, with the symmetric wave function leading to a lower energy eigenvalue. For a repulsive force between the particles the reverse is true. As regards to the wave functions (9.1) one may still think of it in terms of two particles, because the wave functions can be factorized, with the quantum potential becoming a sum of two independent terms

$$-\frac{\hbar^2}{2m}\frac{\nabla^2\sqrt{\underline{\psi}^*\underline{\psi}}}{\sqrt{\underline{\psi}^*\underline{\psi}}} = -\frac{\hbar^2}{2m}\frac{1}{\sqrt{\psi_1^*\psi_1}}\frac{\partial^2\sqrt{\psi_1^*\psi_1}}{\partial x_1^2} - \frac{\hbar^2}{2m}\frac{1}{\sqrt{\psi_2^*\psi_2}}\frac{\partial^2\sqrt{\psi_2^*\psi_2}}{\partial x_2^2}. \tag{9.3}$$

Such a decomposition into parts is not possible for the wave functions (9.2), and it is there then not more possible to think of the two particles which are placed into the well. This, however, is possible by making a 45° rotation in configuration space. Putting

$$\begin{aligned} y &= x_2 + x_1, \\ x &= x_2 - x_1 \end{aligned} \tag{9.4}$$

one obtains the factorized wave functions

$$\begin{aligned} \underline{\psi}_s &= \frac{1}{\sqrt{\pi}} y e^{-(x^2+y^2)/2}, \\ \underline{\psi}_a &= \frac{1}{\sqrt{\pi}} x e^{-(x^2+y^2)/2} \end{aligned} \tag{9.5}$$

for which the quantum potential separates into a sum of two independent terms, one depending only on x and the other one only on y. This means that the addition of a small perturbation in form of an attraction or repulsion between the two particles, transforms them into a new set of two quasiparticles, different from the original particles. With the identification of all particles as quasiparticles of the Planck mass plasma, the abstract notion of configuration space and inseparability into parts disappears, because any many-body system can, in principle, at each point always be expressed as a factorizable wave function of quasiparticles, where the quasiparticle configuration may change from point to point. This can be shown quite generally. For an N-body system, the potential energy can in each point of configuration space be expanded into a Taylor series

$$U = \sum_{k,i}^{N} a_{ki} x_k x_i. \tag{9.6}$$

Together with the kinetic energy

$$T = \sum_{i}^{N} \frac{m_i}{2} \dot{x}_i^2 \tag{9.7}$$

one obtains the Hamilton function $H = T + U$ and from there the many-body Schrödinger equation. Introducing the variables $\sqrt{m_i} x_i = y_i$, one has

$$\begin{aligned} T &= \sum_{i}^{N} \frac{1}{2} \dot{y}_i^2, \\ U &= \sum_{i}^{N} b_{ki} y_k y_i \end{aligned} \tag{9.8}$$

which by a principal axis transformation of U becomes

$$\begin{aligned} T &= \sum_{i}^{N} \frac{1}{2} \dot{z}_i^2, \\ U &= \sum_{i}^{N} \frac{\omega_i^2}{2} z_i^2. \end{aligned} \tag{9.9}$$

Unlike the Schrödinger equation with the potential (9.6), the Schrödinger equation with the potential (9.9) leads to a completely factorizable wave function, with a sum of quantum potentials each depending only on one quasiparticle coordinate. The transformation from (9.8) to (9.9) is used in classical mechanics to obtain the normal modes for a system of coupled oscillators. The quasiparticles into which the many-body wave function can be factorized are then simply the quantized normal modes of the corresponding classical system.

For the particular example of two particles placed in a harmonic oscillator well, the normal modes of the classical mechanical system are those where the particles either move in phase or out of phase by 180°. In quantum mechanics, the first mode corresponds to the symmetric, the second one to the antisymmetric wave function. If it is clear that the quasiparticles representing the symmetric and antisymmetric mode cannot be localized at the position of the particles placed into the well.

10 Wave Function Collapse

The decomposition of a many-body wave function into a factorizable set of quasiparticles, can in the course of an interaction continuously change, but it can also abruptly change if the interaction is strong, in what it is known as the collapse of the wave function going with superluminal speed. It is the wave function collapse which better than anything shows that quantum mechanics is incomplete, a view not only held by Einstein.

Quantum mechanics consists of two parts. The first part is completely deterministic and described by the time evolution of the Schrödinger wave equation. For the second part, the wave function collapse, there is no such theory, except that the probability for the outcome of the collapse is determined by the magnitude of the Schrödinger wave. But there is no deterministic explanation for this probability. In the Copenhagen interpretation the outcome of the wave function collapse is completely stochastic and has no cause. For example, it is claimed that there is no cause why a particular radium atom decays and not all the others. It is understandable why not only many physicists have found this abandoning of the law of causality quite unsatisfactory. It is the Planck mass plasma conjecture which holds out the hope for a deterministic explanation of wave function collapse.

In the Planck mass plasma charge in general, and gravitational charge in particular, has its cause in the zero point oscillations of Planck mass particles bound in quantized vortex filaments. In equilibrium there are $n_\pm = 1/2r_p^3$ positive or negative Planck mass particles per unit volume. In a slightly perturbed configuration one has to replace $n_\pm$ by $n_\pm + n'_\pm$, where $n'_\pm$ are the density perturbations of the positive and negative masses. The perturbed density distributions lead to a gravitational field obtained from Poisson's equation for the gravitational potential ϕ:

$$\nabla^2\phi = 4\pi G m_p(n'_+ - n'_-). \tag{10.1}$$

It has to be supplemental by the continuity equation:

$$\frac{\partial n'_\pm}{\partial t} + |n_\pm|\nabla\cdot\mathbf{v}_\pm = 0 \tag{10.2}$$

where $\mathbf{v}_\pm$ are the velocity disturbances by the positive and negative mass component of the Planck mass plasma.

Newton's equation for the positive and negative velocities is

$$\frac{\partial\mathbf{v}_\pm}{\partial t} = -\nabla\phi. \tag{10.3}$$

From (10.1) and (10.2) one obtains

$$\nabla^2\frac{\partial\phi}{\partial t} = -\omega_p^2\nabla\cdot(\mathbf{v}_+ - \mathbf{v}_-) \tag{10.4}$$

where $\omega_p^2 = 4\pi G|n_\pm|m_p = 2\pi/t_p^2$. Integrating (10.4) on both sides one obtains up to the curl of a function which is of no interest

$$\nabla \cdot \frac{\partial \phi}{\partial t} = -\omega_p^2 \nabla \cdot (\mathbf{v}_+ - \mathbf{v}_-). \tag{10.5}$$

From (10.3) one obtains

$$\frac{\partial^2}{\partial t^2}(\mathbf{v}_+ + \mathbf{v}_-) = -2\nabla \cdot \frac{\partial \phi}{\partial t}. \tag{10.6}$$

Eliminating $\partial\phi/\partial t$ from (10.5) and (10.6) one finally has

$$\frac{\partial^2}{\partial t^2}(\mathbf{v}_+ + \mathbf{v}_-) = 2\omega_p^2(\mathbf{v}_+ - \mathbf{v}_-). \tag{10.7}$$

Assuming that during a wave function collapse $\mathbf{v}_+ - \mathbf{v}_- = \text{const}$, (10.7) leads to the characteristic collapse time

$$t = \frac{1}{\omega_p}\sqrt{\frac{\mathbf{v}_+ + \mathbf{v}_-}{\mathbf{v}_+ - \mathbf{v}_-}} = \frac{t_p}{\sqrt{2\pi}}\sqrt{\frac{\mathbf{v}_+ + \mathbf{v}_-}{\mathbf{v}_+ - \mathbf{v}_-}}. \tag{10.8}$$

One can see that $t \to 0$ if $\mathbf{v}_- = -\mathbf{v}_+$, which demonstrates that very short collapse times are possible in principle. This, of course, also implies that the wave function collapse can go at much greater velocities than the velocity of light. This, of course, is possible only in the presence of negative masses which in the Planck mass plasma are the cause of quantum mechanics as a consequence of Newtonian mechanics analytically continued to negative masses and the local violation of Newton's third law.

11 Conclusion

Present attempts for the unification of physics make the assumption that the theory of relativity and quantum theory, the two pillars of modern physics, are immutable and final truths. In doing so a high price has to be paid. It is the need to assume the hidden existence of more than the three space dimensions, in contradiction to everyday experience. In various string theories there are six more space dimensions, while in M-theory seven more are needed. It appears that under the restrictions of the theory of relativity and quantum theory, a consistent theory of quantum gravity needed for the unification of the gravitational with the other forces of nature is possible only in more than four space-time dimensions.

The unification attempt offered here tries to go a crucial step beyond, not by taking the theory of relativity and quantum theory for granted, but trying to derive both of these theories from a more fundamental principle. As a guide to find this principle I have chosen the similarity between elementary particle and condensed matter physics.

A theory of relativity without a Minkowski space-time as an element of absolute final truth is possible if Lorentz invariance is a dynamic symmetry, as in the pre-Einstein theory of relativity by Lorentz and Poincare' where all the relativistic effects were explained by true physical deformations of rods and clocks in absolute motion through an aether. There a large number of new possibilities emerge. To restrict them to a minimum we take a clue from high energy physics, which tells us that the strong, the weak, and the electromagnetic interaction unify at an energy, on a logarithmic scale, not too far from the Planck energy.

This indicates that this aether should manifest itself at the Planck scale. The most simple assumption is an aether composed of Planck mass particles, each of them occupying a Planck length volume and interacting only by the Planck force over a Planck length. In addition, taking a clue from condensed matter physics it is then conjectured that this aether should resemble a plasma with as many positive as negative mass Planck mass particles. To keep this Planck mass plasma stable, the Planck force between equal Planck mass particles must be repulsive but attractive between unequal Planck mass particles. This then automatically leads to Heisenberg's uncertainty principle for the Planck mass particles, and from there directly to quantum mechanics, with the theory of relativity recovered by the wave-like disturbances propagating through this aether with the velocity of light.

It is quite uncertain if this specific model can survive all the tests required for a viable alternative to string theory, but it appears more plausible than a string theory in higher dimensions. It nevertheless has some intriguing similarities with string theory. The most notable is the topological similarity of vortices and strings, making one wonder if strings are misunderstood vortices. And the need for supersymmetry in string theory may be the misunderstood requirement of negative masses.

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